1. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

a) $t=3,\ H=2.35$

$t=6,\ H=3.28$

$$\text{gradient} = \frac{H_2-H_1}{t_2-t_1} = \frac{3.28-2.35}{6-3}$$

$$= 0.31$$

so ... $H-H_0=m(t-t_0)$

$$H-2.35=0.31(t-3)$$

$$\boxed{H=0.31t+1.42}$$

b) $t=0$: $H=0.31(0)+1.42 = 1.42\text{m} = 142\text{cm}$

from our model we can see at $t=0$, $H=142. \simeq 140.$

So this fact does support our linear model!

2. In 1997 the average CO_2 emissions of new cars in the UK was 190 g/km.

In 2005 the average CO_2 emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO_2 emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n.

(3)

In 2016 the average CO_2 emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

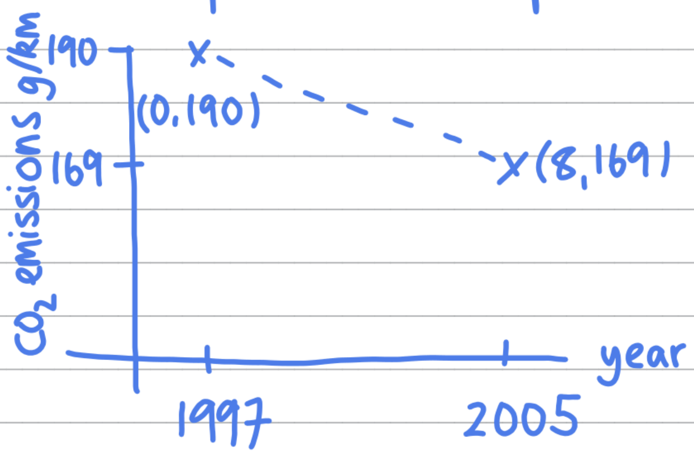
a) linear model: form $A = mn + c$

function of number of years

it can help to visualise problem:

let 1997 be $n = 0$

let 2005 be $n = 8$



so we want equation of dotted line

gradient: $\frac{190 - 169}{-8} = -2.625$

intercept: $190 = c$

so $A = -2.625n + 190$

b) given new data point, we need to see how it compares with model's prediction:

Question continued

$A = -2.625 \times 19 + 190$

$= 140.125 \text{ g/km}$

$140.125 \gg 120 \Rightarrow$ the model overestimates A & so is not suitable

(Total for Question is 6 marks)

3.

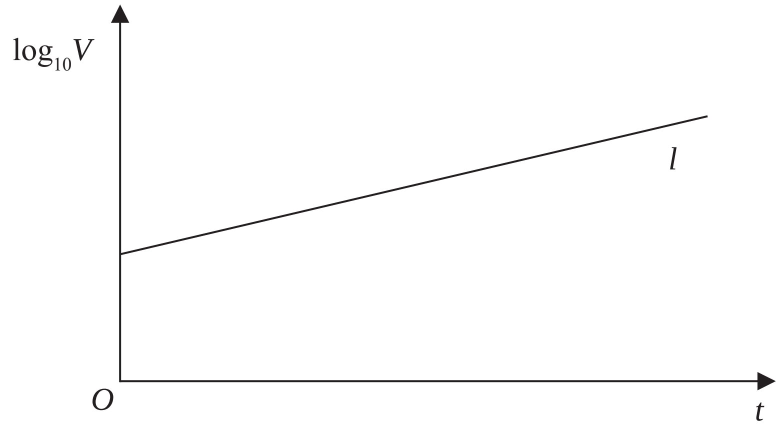


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

a) $p = 10^{4.8}$ $\quad$ $q = 10^{0.05}$

$= 63095.7$ $\quad$ $= 1.122018$

≈ 63100 $\quad$ ≈ 1.122

bi) value of painting on 1st January 1980

ii) The proportional increase of the value each year

c) $2010 - 1980 = 30$

$\log_{10} V = 0.05(30) + 4.8$

$V = 10^{6.3}$

$= 1995262$

$\approx £2000000$

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